November 2017 subject reports

Mathematics HL

Overall grade boundaries

Discrete

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 27	28 - 40	41 - 53	54 - 66	67 - 78	79 - 100

Calculus

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 39	40 - 53	54 - 66	67 - 78	79 - 100

Sets, relations and groups

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 13	14 - 26	27 - 40	41 - 53	54 - 66	67 - 78	79 - 100

Statistics and probability

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 41	42 - 54	55 - 66	67 - 78	79 - 100

Higher level internal assessment

Component grade boundaries

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

The range and suitability of the work submitted

Although a number of candidates chose a good variety of interesting topics, some were favourite modelling topics, such as the “SIR model and Ebola”, or “Travelling salesman problem”. A good number of candidates also presented research reports of well established mathematics. These often reflected plenty of research and the explorations were also well explained. Unfortunately, these types of explorations do not neatly fit all the criteria and often end up scoring less than expected. Problems occur when weaker candidates choose topics that are well beyond their ability to understand the mathematics involved. It is important that candidates are told that they need to think independently and creatively and present mathematical ideas from their own perspective. Some explorations still present statements, formulae, and images simply taken from online sources.

With on screen marking some explorations were difficult to read. Teachers should annotate student work with a red pen, and the explorations should be scanned in colour for uploading. Students should also be aware that shading in pencil is often not picked up in a scan and this makes their work very difficult to moderate. Students and teachers could check scans before uploading to avoid this.

Candidate performance against each criterion

A: The majority of explorations seen were well organised with introduction, an aim, a rationale, a body and a conclusion. Coherence was more variable. Some schools presented explorations with tables of content and a research question. Neither of these are appropriate for an Exploration. It is also important that candidates stay focused on the aim that was chosen. Work that is interesting in itself but is not directly related to the aim is likely to cause lack of coherence and make the exploration not concise. Many explorations were far too long, being in excess of 20 pages single spaced writing. Candidates should be reminded that work should be presented double spaced and with a reasonable font style and size e.g. Arial 12.

B: Most candidates were careful to define variables and key terms and most graphs were appropriately labelled. However, in many cases, where the candidates wrote about topics outside the syllabus, or modelled a situation from another discipline e.g. Physics or Economics, key terms were not defined. Candidates need to be reminded that any new terms, variables or topics need to be carefully introduced. Prior knowledge should not be assumed. Careless use of wrong terminology such as “plug in” for “substitute” should be discouraged.

C: Personal engagement is frequently at a low level in the explorations that are mere “research reports”. Explorations are generally better if the mathematics is used to solve a problem rather than a problem being found to illustrate some mathematics. Although a candidate may show

International Baccalaureate® Baccalauréat International Bachillerato Internacional

some personal engagement by learning new mathematics or teaching themselves a topic, this alone does not justify top levels being awarded in this criterion.

D: Good reflection is usually seen throughout the exploration and drives its development. Unfortunately when students present a research report they are left with little opportunity to reflect critically on the mathematics used. Candidates should be encouraged to reflect on their work frequently and to report on their emergent thinking and how this thinking led them to the next part of their exploration. In some explorations the reflection was a mere summary of the work done as a conclusion. Candidates should be reminded that this does not constitute reflection, and will not allow them to obtain any marks for this criterion.

E: This criterion requires the use of mathematics and not the mere reporting on theorems and their proofs.

The mathematical content varied greatly, from very basic to extensions well beyond the HL core. It was evident from the teacher notes that some candidates are being encouraged to write explorations on mathematics that is beyond the HL course. The choice of topic is key to performing well in this criterion. Teachers need to provide proper guidance to allow their students to choose topics that they can do justice to. Those who choose something too simplistic (e.g. data collection with mere arithmetic, and GDC-based regression analysis) or too advanced will invariably fail to obtain a good score here. In some cases candidates seemed to approach a straight forward problem in a complex way in order to demonstrate some mathematics. This approach may suggest that there is little understanding.

Recommendations for the teaching of future candidates

Teachers should ensure that the exploration is done after a reasonable amount of HL topics are covered. Some explorations indicated that students needed more guidance during the IA process. At the time of choosing topics, teachers need to be fully involved in guiding their students to choose topics that are appropriate for their exploration, allowing them to achieve appropriate levels in each of the criteria and avoiding pitfalls. Once the exploration is submitted the teacher must show evidence of marking with annotations and comments on the student work. The use of “callouts” with a summary at the beginning or end of the work is not helpful for moderators. Annotations should be visible where they are intended. Evidence that the mathematics has been checked for errors should also be present. This can take the form of tick marks next to the work.

Further comments

Marking explorations, appropriate annotations on students' written responses and processes to help make a good choice of topic need to remain an important part of workshops. It remains important that teachers are able to effectively communicate their reasoning behind their choice of levels of the criteria for moderators to be able to confirm their marking. Some samples were marked down because of errors, noted by the moderators, which were not picked up by the original marker.

International Baccalaureate® Baccalauréat International Bachillerato Internacional

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 12	13 - 25	26 - 37	38 - 50	51 - 62	63 - 75	76 - 100

General comments

On the surface, this paper appeared a difficult one. Nevertheless, there were a good number of high scoring scripts seen. Perhaps unsurprisingly, the first part of question 10 caused some problems, while well-presented and correct solutions to 11d were rarely seen.

The areas of the programme and examination which appeared difficult for the candidates

Geometrical properties of vectors. De Moivre’s theorem. Counting principles.

The areas of the programme and examination in which candidates appeared well prepared

Algebra. Calculus. Binomial series. Curve sketching

The strengths and weaknesses of the candidates in the treatment of individual questions

Section A

Question 1

This provided a generally easy start for many candidates. Most candidates scores either 4 or 5 marks, with some failing to realise that x cannot be negative, and thus failing to gain the final mark.

Question 2

This was another generally straightforward question that posed few problems. A small number of candidates reached the stage where $82\lambda = 41$ before stating incorrectly that $\lambda = 2$. A similarly small number seemed to insist on applying the cross product somehow to determine $\overrightarrow{AB}$, and as a consequence, gained few, if any marks.

Question 3

Part a) was convincingly handled, as was part b). Even those candidates who chose to employ a synthetic division method in either part made few, if any errors.

Question 4

This was answered very well by the vast majority. Again, careless errors seen in calculating the correct coefficient were few and far between, although some did state the final coefficient to be -84 , mainly due to not squaring the previous '-2' correctly.

Question 5

This seemed to be the stage in the paper where most candidates appeared to start having difficulties. Integration by parts was generally handled capably. However, a number chose to ignore the initial coefficient of 10 after becoming distracted by the integration by parts technique.

A number of candidates also substituted $t=\frac{1}{2}$ into their integrated expression, but failed to deal with the lower limit. It was not particularly clear whether it was the kinematic context or the use of limits in integration by parts that was the underlying issue for the majority.

Question 6

Part a) was generally answered very well, with only a very small number of scripts seen where the sketch appeared in the incorrect quadrants.

In part b), several methods were available to the candidates, with most choosing to form two separate equations and solve. As a result, correct critical values were often (but not always) obtained. A number of candidates, having found $x=-3$ and $x=1$, proceeded to choose the complement of the correct solution.

Question 7

This was perhaps a longer question on implicit differentiation than a number of candidates seemed used to. The correct expression for $\frac{\mathrm{dy}}{\mathrm{dx}}$ was often obtained, as was the equation $y^2-x=0$. A number of candidates stopped at this point, though those that knew what to do with this equation, generally went on to confidently secure the final marks.

Question 8

This proved to be a good discriminator. A number of candidates scored full marks, while a similar number also scored 0 or 1 marks. While a number of methods were open to candidates, the intention was that they would employ the use of De Movire's theorem. For those that were confident in tackling these types of question, these candidates were able to determine the final roots quickly and without problem. For others, various ‘arguments’ were seen within their expressions for $z+2\mathrm{i}$, while some seemed to give up before even finding three distinct roots, correct or not.

Section B

Question 9

Parts a-c comprised a standard type of question with which most candidates seemed familiar. However, even with apparent scaffolding to help candidates, a good number found this question challenging, especially part c). A number resorted to using the given value for μ rather than equating two pre-determined equations. Part d) was successfully answered by the vast majority of candidates. Part e) caused the greatest issues. A number scored poorly despite obtaining the correct answer, usually due to an assumption that they were dealing with right-angled triangles when determining their areas. It was intended that candidates should use the cross product in an unfamiliar setting, and it was therefore pleasing to see a small number of candidates gain the correct $k=26$ through this method, making the question a good discriminator at the higher end.

Question 10

Part a) inevitably proved to be difficult for many candidates, with some possibly relying on guesswork or giving ‘methods’ that were, at the very least, lacking in clarity. A number convincingly listed all 24 possible options for the cards and therefore found it an easy task to identify the required 9. Another method often seen was a probability distribution table, with candidates giving the correct respective probabilities for 1, 2 and 4 matches. While being a challenging question, this did provide some candidates with an opportunity to show their reasoning and as well as presentational skills in mathematics. Part b) was inevitably answered correctly, especially with the probability to be used, given as a ‘show that’ in part a). A number of candidates however found problems with part bii, often resorting to using an inappropriate formula, or even failing to evaluate their fraction correctly. Careless answers of $\frac{750}{32}$ or $\frac{375}{64}$ were sometimes seen.

Question 11

Part a) was usually answered correctly, though some justifications were only partially complete.

Part b) again separated those candidates who could use an inductive technique from those who could not. In essence, this was a fairly straightforward type of induction, and a significant number of candidates gained 7 or 8 marks here. A number reached the stage of writing down the correct expression for the case $n=k+1$, but then went somewhat adrift when mistakenly trying to apply the result $\cos 2x = 1-2\sin^2 x$ (or similar), to their $\cos^{2k+1} x$ term.

Part c) was generally answered well, albeit with the occasional slip seen when differentiating. A number of candidates used the product rule here rather than the intended quotient rule, though usually managed to obtain a correct expression for the derivative.

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Perhaps it was due to time constraints, but it was rare to see some well-presented mathematics leading to the result $f_n{}'\left(\frac{\pi}{4}\right) = 2$ in part d). While some proceeded to obtain the correct result, it was felt in several scripts that candidates' presentation and arguments could have been improved in this part of the question.

Recommendations and guidance for the teaching of future candidates

Many candidates seemed less than adept at recognising a potential application of De Moivre's theorem in question 8. Sufficient practice in this area of the syllabus would be beneficial.

Similarly, question 9 continued to illustrate candidates' general difficulties and a lack of confidence in using vectors; this applies to both familiar and unfamiliar contexts.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 28	29 - 39	40 - 53	54 - 66	67 - 80	81 - 100

The areas of the programme and examination which appeared difficult for the candidates

- Applying knowledge of geometric series to problems involving compound interest combined with deposits and withdrawals.
- Solving problems using counting principles.
- Using trigonometric identities to transform trigonometric equations.
- Finding an integral using a trigonometric substitution.
- Identifying the key features of a graph, using a graphical display calculator, to produce a reasonable sketch.
- Understanding the relationship between the graph of a function and the graph of a derivative.
- Solving trigonometric problems in 3 dimensions using angles of depression.
- Using a graphical display calculator to solve a system of equations efficiently was not universally seen.
- Appreciating the difference between the command terms "Show that" and "Verify" as detailed in the guide. In "Show that" questions, the use of a calculator is generally not required.

The areas of the programme and examination in which candidates appeared well prepared

- Using knowledge of sets and Venn diagrams to solve problems involving the probabilities of combined events.
- Setting up linear simultaneous equations to solve simple problems in context, often demonstrating efficient use of the graphical display calculator.
- Using the formula for the area of a sector to solve problems in context.
- Solving problems involving the roots of a quadratic equation.
- Finding the mean and standard deviation of a normal distribution, given information about associated probabilities.
- Finding a volume of revolution.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

Generally very well done, mostly by candidates who used a calculator after first writing the appropriate equations. Most interpreted the question correctly, but those who tried to solve without the calculator rarely obtained correct solutions, and spent too long on this question. Some candidates left their answers as fractions, which was not appropriate in the context of the question.

Question 2

Generally well done, where students used appropriate formulae. Many successfully used a Venn diagram to support their solutions. However, a number of students assumed independence without proof in parts a) b), and even occasionally in part c), resulting in a circular argument which scored no marks. Some reasonable solutions to part c) were incomplete and lacking a formal conclusion.

Question 3

Generally very well done. However, a few students attempted to use the sector area formula from the formula book without converting from degrees to radians. Some also attempted to use the formula for the area of a triangle rather than the formula for the area of a sector.

Question 4

Generally well done, but with a wide range of performance. Some candidates failed to use the inverse normal function and others misinterpreted the information given and reversed the inequality signs in their probability statements, resulting in z-values of the incorrect sign. Most used their calculator efficiently throughout, although some of these showed very few steps. On the other hand, some spent too long on manual methods for solving simultaneous equations in μ and σ.

International Baccalaureate® Baccalauréat International Bachillerato Internacional

Question 5

This was well done by those who understood what an angle of depression is, and by those who appreciated that the diagram was not intended to represent a three dimensional situation. The most efficient solution involved the use of tangent and the cosine rule, and it was disappointing to see candidates using the sine rule in a right angled triangle. Some candidates tried to solve the problem completely without context, interpreting the diagram in 2 dimensions.

Question 6

Part a) was often well done, although some struggled to appreciate that $P(X \geq 1) = 1 - P(X=0)$. Part b) was solved using either a Poisson or a binomial distribution, both equally successful. A surprising number of candidates suggested that there are 48 weeks in a year.

Question 7

This question was often well done, using a variety of methods including sums and products of roots, the use of the quadratic formula and comparing coefficients. Those who used the factor theorem were also largely successful, although it was rare to justify why the solution where both roots are 0 should be rejected.

Question 8

Only completed successfully by a small number of students, but it was pleasing to see a number of confident solutions with a variety of approaches to using the given substitution. Most students attempted the question, and there was evidence that the candidates understood the need to show all their steps carefully including the final step of rearranging the substitution to obtain an expression for θ in terms of x.

Question 9

Part a) was usually well understood but many candidates were unable to clarify their thinking and identify the situations to be considered in order to solve parts b) and c). Many successful candidates included helpful diagrams to support their solutions.

Question 10

In part a)i), a large number of candidates failed to understand the nature of the "Show that" command term, where a verification using a calculator is not appropriate. Even those who scored marks for correctly differentiating, and setting their derivative equal to 0, still lost the final mark for resorting to calculator verification as their final step. Part a)ii, by contrast, was solved most efficiently using the calculator to find the minimum value of the function.

In part b), the correct domain and the asymptotic behaviour were sometimes not clearly shown. However, this question was generally well answered.

In part c), many candidates demonstrated an understanding of the relationship between the gradient of the curve and the gradient of the normal, but it was not common to see efficient use of the calculator to find the coordinates of the required point. Some candidates did not try to

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

find the y-coordinate of the point. Recommended calculators allow candidates to solve problems using the graph of the derivative, without the need for an algebraic expression for the derivative.

Part d) was often easily solved, although some candidates with the correct expression failed to enter it correctly into their calculators, often because their calculator was in degrees mode. Some also failed to square the function, finding an area rather than a volume.

Question 11

In part a), many differentiated correctly but many failed to find the local maximum on the graph of the derivative, or to show the correct domain clearly, and sketches were often difficult to read. However, there were a surprising number of candidates who struggled to use the chain rule correctly. Candidates struggled to understand that the point of inflexion on a graph corresponds to a turning point on the graph of the derivative.

Part b) was either done well or not at all, with many candidates unclear on how to use the relationships between trigonometric ratios to transform an equation.

Part c) was either done well or not at all, and there were various creative approaches which allowed candidates to use the answer given in part b) to solve this part of the problem.

Question 12

Parts a), c) and d)ii) were often well done, with many candidates able to use appropriate formulae to solve relatively simple problems involving geometric series. A common mistake was to use simple interest instead of compound interest in part a). In part c), some candidates equated the expression in b) to $150 000 instead of using the answer found in part a).

In part b), candidates were often given the benefit of the doubt despite minimal reasoning shown. Few were able to demonstrate clearly that they understood why a geometric series was appropriate.

In part d)ii), more explanation was required, and many candidates simply re-expressed the given answer for small values of n, without explanation. This did not demonstrate adequate reasoning to be awarded full marks.

Recommendations and guidance for the teaching of future candidates

- Ensure that all parts of the syllabus have been taught; there still seemed to be a group of candidates who were lacking knowledge of probability despite being strong in other areas. There were also many who were not familiar with angles of depression and trigonometric problems in three dimensions. It was not always clear that students had experience of problems involving compound interest combined with deposits or withdrawals.

- Ensure that candidates know how to use their graphical display calculator efficiently.

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Examples include solving systems of linear equations, using degrees and radians modes appropriately, knowing how to graph the derivative without differentiating by hand, and adjusting the window to the domain of a function.

- Explicitly teach how to respond to the command term "Show that", including logical steps and a concluding statement. Candidates are encouraged to use worded statements where appropriate to support their arguments. The guide states that these questions "do not generally require the use of calculator".

- Highlight that the sector area and arc length formulae given in the formula book only apply to angles given in radians.

- Encourage candidates to write down probability statements and equations when solving problems involving a normal distribution.

- Encourage candidates to explain their thinking when solving problems involving counting principles. Diagrams can be very helpful.

- Insist that students practise drawing clear, labelled sketches of graphs. The command term "Sketch" requires " a general idea of the required shape of relationship, and should include relevant features."

Higher level paper three discrete

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 8	9 - 16	17 - 22	23 - 28	29 - 34	35 - 40	41 - 50

General comments

For the graph theory question there is absolutely no need to draw the graphs on graph paper.

This is not a paper that a candidate can just walk into and expect to do well in, if they have not studied the syllabus and are not aware of the methods and notation.

The areas of the programme and examination which appeared difficult for the candidates

There were too many candidates who had not prepared sufficiently for this paper. For example not knowing how to solve a recurrence relation.

The areas of the programme and examination in which candidates appeared well prepared

Candidates could generally perform the graph algorithms and had a method of solving linear congruences. Some candidates were good on the graphs and poor on the integers but also vice versa.

The strengths and weaknesses of the candidates in the treatment of individual questions

Q1.

(a) The graph was normally drawn correctly.

(b) There were many good answers. Mistakes included forgetting to go back to D. There was still confusion with twice a minimum spanning tree which is not on the syllabus. A systematic layout of work helped the examiner allocate marks.

(c) Reasonably well done. Not all candidates explained sufficiently the method that they were using. Some candidates just applied the nearest-neighbour algorithm to the subgraph. A few candidates wasted time by considering all the cases of deleting a vertex rather than just C.

Q2.

(a) This tended to either gain full marks with the candidate knowing the method or no marks with the candidate aimlessly working out the first few terms.

(b) This distinguished good candidates who saw that it followed from applying Fermat's Little Theorem even if they did often miss the reasoning mark. Most candidates did not make a good attempt and there were doomed attempts at induction on primes.

Q3.

(a) The graph was usually drawn correctly. There were enough scripts that gave the standard proof but not all had the reasoning mark for there being no triangles. There were also many attempts that were just waffle rather than a proof by contradiction.

(b) Either done quickly and confidently or again more waffle.

(c) If they thought about the number of edges in a complete graph they were fine. Candidates without the knowledge tried to fudge the answer.

Q4.

(a) The 2 reasoning marks were not always picked up although the candidate could obtain the answer.

International Baccalaureate® Baccalauréat International Bachillerato Internacional

(b) The answer was sometimes shown with insufficient explanation. With the 3 methods shown in the mark-scheme, the first one shows a reasoning systematic approach. Experience shows that the 3rd method is often learned by rote and applied (often wrongly) with little understanding. For these reasons, although allowed, it is not recommended by me.

Q5

(a) Most candidates had the correct answer if they understood base algebra.

(b) Most answers correct but a few surprises on understanding of primes

(c) The clear reasoning and elimination of the other possibilities was beyond all but the best candidates. The answer was sometimes reached without good justification. Some explanations showed confused understanding.

Recommendations and guidance for the teaching of future candidates

The rubrics at the top of page 2 should be known beforehand. Especially : “Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations.” Answers that just chose a particular numerical value (e.g. in question 3) are never going to gain the marks. A full trial exam is essential and needs to be correctively marked to point out such logical flaws.

Higher level paper three calculus

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 14	15 - 21	22 - 27	28 - 33	34 - 39	40 - 50

General comments

This exam was considered a reasonably accessible test of a candidate’s knowledge of calculus. Although a good percentage of candidates demonstrated sound knowledge of the syllabus, it was also apparent that a significant number of candidates were not sufficiently prepared and performed poorly.

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

The areas of the programme and examination which appeared difficult for the candidates

Using core differential and integral calculus skills and techniques, e.g. applying the chain rule and recognising when to use integration by substitution.

Confusing the limit comparison test with the comparison test and using incorrect notation when reasoning why a series is convergent.

Calculating the interval of convergence of a power series.

Erroneously stating that $\cos\left(\sqrt{5\pi}\right) = -1$ (confusing this with $\cos(5\pi) = -1$).

Using a GDC to solve an equation by graphical means.

Interpreting the mean value theorem and illustrating it graphically.

Proving a property of a general Maclaurin series for $f(x)$.

The areas of the programme and examination in which candidates appeared well prepared

Understanding the requirements for a function and its derivative to be continuous at $x = x_0$.

Demonstrating that a first-order linear differential equation has an integrating factor.

Using the ratio test on a power series.

Using a general differential equation that satisfies a function f and its derivatives to show that $f(x)$ has a given Maclaurin series.

Evaluating a limit (0/0 indeterminate form) using L'Hôpital's rule or a Maclaurin series.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1:

Question 1 was generally well done with most candidates understanding the requirements for a function and its derivative to be continuous at $x = x_0$. Some candidates approached differentiability from first principles. After correctly finding $a = 2$, a small number of candidates made a careless error when solving $2 + b = -1$ for b.

International Baccalaureate® Baccalauréat International Bachillerato Internacional

Question 2:

Part (a), a show that question, was reasonably well done by a large number of candidates. Successful candidates used either integration by substitution or integration by recognition and then algebraic simplification involving logarithms and exponentials to show that $\sqrt{x^2+1}$ was the integrating factor. Some candidates 'jumped directly' from $e^{\int \frac{x}{x^2+1}dx}$ to $\left(x^2+1\right)^{\frac{1}{2}}$ while others appeared to attempt to work ('fudge') back from the integrating factor. Some candidates did not show all the steps required to obtain full marks.

Due largely to poor core calculus skills and knowledge, part (b) was not as well done as anticipated. A significant proportion of candidates were able to correctly obtain $y\sqrt{x^2+1}=\int x\sqrt{x^2+1}\,dx$. However, a number of candidates then attempted to use integration by parts rather than integration by substitution. Having obtained $y\sqrt{x^2+1}=\frac{1}{3}\left(x^2+1\right)^{\frac{3}{2}}+C$, a few candidates correctly determined that $C=\frac{2}{3}$ but then gave $y=\frac{1}{3}\left(x^2+1\right)+\frac{2}{3}$ as their final answer. Other errors included neglecting to specify an arbitrary constant and not attempting to evaluate the arbitrary constant.

Question 3:

Part (a) was not as well done as anticipated. Many candidates attempted to use the comparison test instead of the limit comparison test as stated in the question. A significant number of candidates incorrectly stated that $\frac{1}{n^2}$ was convergent. Such a statement, missing the summation symbol, relates to the generating sequence rather than to the series. A few candidates confused limit notation with series notation while some others attempted to use a 'ratio test'.

In part (b), a significant number of candidates were able to use the ratio test to determine that $\sum_{n=1}^{\infty}\frac{(x-3)^n}{n^2+2}$ converges if $|x-3|<1$. Of these, a number of candidates did not go on to consider the endpoints $x=2$ and $x=4$. Most candidates that did consider $x=4$ recognised that $\sum_{n=1}^{\infty}\frac{1}{n^2+2}$ is convergent from part (a). Often it was the best candidates who were able to demonstrate that when $x=2$, $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2+2}$ is convergent and thus determining $2\le x\le 4$ as the required interval of convergence. Surprisingly, some candidates omitted the modulus sign or made an error when attempting to solve $|x-3|<1$ for x while others laboured determining that $\lim_{n\to\infty}\frac{n^2+2}{(n+1)^2+2}=1$.

Question 4:

In part (a), a significant number of candidates either exhibited poor use of a GDC or poor core differentiation skills or both. When attempting to find $g'(x)$, quite a few candidates did not apply both the product and chain rule, often only applying the product rule. A few candidates had a sign error in their expression for $g'(x)$. When calculating the gradient of the chord, a number of candidates erroneously stated that $\cos\left(\sqrt{5\pi}\right)=-1$, confusing this with $\cos(5\pi)=-1$. Quite a few candidates were able to correctly state the gradient of the chord as $\frac{g(5\pi)-g(0)}{5\pi}$ but were then seemingly unable to enter this expression into their GDC correctly to obtain $-0.6809...$. A few candidates attempted to solve $\cos\left(\sqrt{c}\right)-\frac{\sqrt{c}\sin\left(\sqrt{c}\right)}{2}=\cos\left(\sqrt{5\pi}\right)$ (or equivalent) for c without a GDC while a number of others had the correct equation but obtained incorrect answers from their GDC with some of these having their GDC set to degrees.

Part (b), which was designed to test a candidate's ability to interpret the mean value theorem and illustrate it graphically, was done a little better than anticipated. A large number of candidates were able to correctly sketch the graph of $y=g(x)$ on the interval $[0,5\pi]$. A surprising number of candidates omitted the chord to which the tangents were to be shown to be parallel to demonstrate the mean value theorem.

Question 5:

Part (a), which tested chain rule differentiation, was generally well done although a few candidates treated the function as a product. In part (b), a good majority of candidates were able to show that $f^{(n+2)}(0)=\left(n^2-p^2\right)f^{(n)}(0)$. In part (c), a good number of candidates were able to obtain the first 3 marks. To obtain the last mark, candidates needed to link the third and fifth derivatives to the first and third derivatives respectively or state and use the general Maclaurin series. As the Maclaurin series for $f(x)$ was given, candidates needed to convince the examiner that they knew what they were doing. In part (d), a significant number of candidates were able to evaluate the limit (0/0 indeterminate form) either by applying L'Hôpital's rule or using the given Maclaurin series. Some candidates cleverly exploited the link with part (a). Part (e) proved to be beyond the capabilities of all but the strongest candidates.

Recommendations and guidance for the teaching of future candidates

It is imperative that students who take the Calculus option have a good foundation in core calculus skills and knowledge.

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Teachers should continue to emphasize the need to use correct notation when representing a series. Students need to be dissuaded from making statements such as '$\frac{1}{n^2}$ is convergent '. For example, in question 3(a), a number of candidates lost one mark because they failed to use correct series notation.

More attention should be directed to the difference between the comparison test and the limit comparison test. Sigma notation is required when reference is made to a series.

Teachers must emphasize the need to show appropriate reasoning and clear methods/steps leading to the correct answer in a "show that" question.

Teachers should advise candidates to read the questions carefully to improve the quality of response to the problem and write legibly as this will greatly help examiners with their marking.

Higher level paper three sets, relations and groups

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 7	8 - 14	15 - 23	24 - 28	29 - 33	34 - 38	39 - 50

General comments

Although this paper was accessible, surprisingly, some candidates showed lack of familiarity with some basic ideas contained in this option. A significant proportion of candidates were very careless in the process of manipulation. This was especially evident in question 3.

The areas of the programme and examination which appeared difficult for the candidates

Candidates had some difficulty in applying definitions they clearly knew to specific examples in 'show that' and 'prove that' questions. Although the definition of an equivalence relation and the properties of groups were well known, at times the definitions and properties were not interpreted correctly within the given examples.

Many candidates showed difficulties in using correct mathematical notation, particularly as pertains to equivalence relations and homomorphisms. Some candidates also did not know how to determine the symmetric difference of two given sets.

Many candidates showed some difficulty in determining equivalence classes of a given equivalence relation.

International Baccalaureate® Baccalauréat International Bachillerato Internacional

Most candidates were either unfamiliar with the definition of coset or showed little understanding of its meaning.

The areas of the programme and examination in which candidates appeared well prepared

Candidates had good awareness of key definitions contained in this option. They generally showed good ability in answering questions on set operations. They were familiar with properties of equivalence relations, definition of homomorphism and order of an element of a group, and could satisfactorily show that a given relation on a set was an equivalence relation, and a binary operation on a given set satisfied the commutative and associative properties.

The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1

a) (i) Most candidates had an idea about what order of an element meant and made a good attempt to this question. Several candidates however provided answers that showed no understanding of the concept of finite group.

(ii) Most candidates said that the group was cyclic, but the reasons provided were very often incorrect. Many candidates wasted time proving that the structure was indeed a group.

b) Surprisingly many candidate´s answers were subsets of *G* that did not include the identity.

c) Very few candidates knew the meaning of coset. The candidates that knew it answered this question well.

Question 2

a) Candidates answered this question well using the most usual configuration of a Venn diagram with 2 or 3 intersecting sets. In a few cases candidates used configurations with non-intersecting sets A and C.

b) (i) Apart from minor mistakes, most candidates were successful answering this question. In a few cases candidates wasted time trying to simplify the expressions instead of using the sets provided to verify that the equality did not hold in the case provided.

(ii) This question was not well answered, and several misconceptions were identified. For example, candidates referred to distributivity as a property of the sets rather than of the operation. Also in most statements about distributivity just one operation was referred.

Question 3

a) This question was poorly answered. Most candidates could just name the properties that they need to prove, and even these were sometimes incorrect. Then the list of errors included ignoring the definition of the relationship provided and the implications relating what is assumed and what is to be proved for symmetric and transitive.

(b) Very few candidates scored full marks in this part. The most common omission was the point (1,2) on the graph; other candidates ignored the set where the relation was defined. Many candidates assumed that the relation was defined in □ ×□ .

Question 4

(a) This question was poorly answered although in many cases candidates knew what they needed to show.

(b) (i) This was the part of question 4 that candidates answered well although a few attempts to establish commutativity using examples were seen;

(ii) associativity was well attempted, but many candidates made mistakes while expanding the expressions.

(c) Many candidates were successful testing left or right identity but very few provided a justification for the existence of both.

(d) This part of the question was not well answered. Some candidates did find an expression for the inverse of each element but did not show it belonged to *S*. In many cases candidates showed confusion with notation for inverses and replaced a^{-1} by 1/a and proceeded to obtain an incorrect answer.

Question 5

This question was poorly attempted.

(a) Many candidates just quoted the property that they were required to prove. In other cases, candidates tried to use other properties but in a very few cases a serious attempt to establish the result from what was given were made.

(b) Just a very small number of candidates seemed aware of what was being asked. In many cases, candidates just quoted results. In many other cases candidates wrote incorrect statements showing no understanding of this part of the option.

Recommendations and guidance for the teaching of future candidates

Candidates should be exposed to different kinds of problems in which they need to interpret the properties of groups and equivalence relations, including how to find equivalence classes.

The use of correct communication and notation should be stressed, as well as all steps necessary in justifying their conclusions.

Candidates should be made aware of the need of being more rigorous in setting out proofs. Be harsh in scoring the details of equivalence relation proofs so that candidates learn the importance of precision. Make candidates aware of circular reasoning.

Expose candidates to more examples of equivalence relations and corresponding equivalence classes.

Higher level paper three statistics and probability

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 10	11 - 20	21 - 26	27 - 31	32 - 35	36 - 40	41 - 50

The areas of the programme and examination which appeared difficult for the candidates

Many candidates were unable to express correctly the relationship between the probability density function and the cumulative distribution function of a random variable.

Many candidates were unable to apply the conditional probability formula in the context of probability distributions.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are confident when using a GDC to carry out statistical tests.

Most candidates are able to calculate percentiles using the cumulative distribution function.

Most candidates understand the notion of unbiased estimators and most efficient estimators.

The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 – Rather surprisingly, (a) was one of the worst answered question parts on the paper. The majority of candidates found $F(t)$ as the indefinite integral of $f(t)$ which in this case gave the correct answer but nevertheless was not given full credit. Full credit was only given to candidates who used the correct limits or who introduced a constant of integration which was then evaluated using an appropriate boundary condition. Even many of the candidates who used limits stated that $F(t)=\int_0^t f(t)\,\mathrm{d}t$. This use of a symbol to denote both the upper limit of integration and the dummy variable of integration is not just poor notation, it is mathematically incorrect. It was condoned on this occasion but this may not always be the case. Part (b) was well answered in general. The expectation in (b)(ii) was that candidates would use their GDC to locate the point of intersection of $y=F(t)$ and $y=0.75$. However some candidates factorised $F(t)$ into its four linear factors leading to the four roots $\pm\sqrt{2},\pm\sqrt{6}$. Most candidates then chose $\sqrt{2}$ as the only root lying in $[0,2]$ but those few who gave all the roots as possible values of the median were of course not given the final accuracy mark.

Q2 – Some candidates chose to calculate the mean and variance estimates using the formulae which sometimes led to arithmetic errors and division by n instead of $n-1$ to estimate the variance was often seen. Candidates who used the GDC to give the mean and variance estimates sometimes squared the wrong GDC 'standard deviation'. It is worth noting that it is the larger of the two which should be squared. Most candidates used the calculator software to carry out the *t*-test although a small minority incorrectly used a Z-test. Some candidates calculated the *t*-value using the formula and then found the *p*-value by using the *t*-distribution cumulative probability function on the calculator. This is of course a valid method but it is more time consuming than intended and not to be recommended.

Q3 – Part (a) was well answered in general. It was pleasing to note that only a small minority failed to realise that an expectation needed to be taken. In (b)(i), some candidates failed to give a convincing 'show that' solution. Candidates were expected to make a positive statement along the lines that, in sampling, $\mathrm{Var}(\overline{X})=\frac{\sigma^2}{n}$ but many failed to do this. In (b)(ii), many candidates were successful in finding the minimum variance estimator using either differentiation or by locating the vertical axis of the quadratic in a. Some candidates, realising that the variance had to be minimised, equated it to zero which of course led to an incorrect result. Candidates who found the correct value of a usually continued successfully to find the minimum variance estimator. Some candidates misread the question thinking that the minimum variance was required instead of the minimum variance estimator but this was not given any credit.

Q4 – Part (a) was correctly answered by almost all the candidates. Solutions to (b) were generally good although some candidates used incorrect degrees of freedom and others carried out a one-tailed test. Partial credit was given for solutions with these errors.

Q5 – Both parts of (a) were correctly answered by almost all the candidates. Many candidates were able to show that the probability generating function of $X+Y$ is $\mathrm{e}^{(\lambda+\mu)(t-1)}$ but some then failed to explain why this result shows that $X+Y$ has a Poisson distribution. Candidates were required to explain explicitly that this is the probability generating function of a Poisson distribution with mean $\lambda+\mu$ and many failed to do that. Some candidates simply differentiated their result and put $t=1$ to show that the mean was $\lambda+\mu$ completely ignoring the distributional aspect. Other candidates wrote that $G_{X+Y}(t)=G_X(t)G_Y(t)$ and then just differentiated this expression and put $t=1$ to show that $\mathrm{E}(X+Y)=\lambda+\mu$ without considering distributions. Some candidates went on to show that the variance was also $\lambda+\mu$ and then stated, incorrectly, that this showed that $X+Y$ has a Poisson distribution.

Most candidates were unable to make much progress in (c)(i).

Even those candidates who began by stating, correctly, that $\mathrm{P}(X=x \mid X+Y=n)=\dfrac{\mathrm{P}(X=x \cap X+Y=n)}{\mathrm{P}(X+Y=n)}$ were often unable to make further progress.

Almost every candidate answered (c)(ii) correctly, independently of any work done in (c)(i).

Recommendations and guidance for the teaching of future candidates

Many candidates seem to be unaware of the instruction on the front of the examination paper which states that 'Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures'. Many candidates lose marks by failing to obey this instruction.

Candidates should be aware that in 'show that' questions, care must be taken to justify fully the steps leading to the final (given) answer.

Candidates should know that the correct way to express the relationship between $f(x)$ and $F(x)$ in the context of probability distributions is $F(x)=\int_{-\infty}^{x} f(u)\mathrm{d}u$.

Candidates should be aware that it is quicker and probably more reliable to carry out statistical tests using the GDC rather than calculating the value of the statistic manually and only then using the GDC to find the *p*-value.

International Baccalaureate® Baccalauréat International Bachillerato Internacional