

Differentiability

Evaluate functions:

$$1. \quad f(x) = \begin{cases} 1-x & x < 1 \\ (1-x)(2-x) & 1 \leq x \leq 2 \\ 3-x & x > 2 \end{cases}$$

Discuss continuity and differentiability of above function.

$$2. \quad f(x) = \begin{cases} x^2 + 3x + a & \text{for } x \leq 1 \\ bx + z & \text{for } x > 1 \end{cases}$$

If above function is differentiable every where find a and b.

$$3. \quad \text{Discuss the differentiability of } f(x) = |\log_e x|, \text{ for } x > 0.$$

$$4. \quad \text{Discuss differentiability of } f(x) = |x - 1| + |x - 2|.$$

$$5. \quad \text{If } f(x) = \begin{cases} x \left(\frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} \right), & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Then show that $f(x)$ is not differentiable at $x = 0$.

$$6. \quad f(x) = \begin{cases} ax(x-1) + b, & x < 1 \\ x-1 & 1 \leq x \leq 3 \\ px^2 + qx + 2 & x > 3 \end{cases}$$

If $f(x)$ is continuous at $x \leq R$ and $f(x)$ is not differentiable at $x = 1$ then find a, b, p and q.

$$7. \quad f(x) = \begin{cases} (x-1)^2 \sin\left(\frac{1}{x-1}\right) - |x| & x \neq 1 \\ -1 & x = 1 \end{cases}$$

Find all points where $f(x)$ is differentiable.

$$8. \quad \text{Show that } f(x) = |x| \text{ is not differentiable at } x = 0.$$

$$9. \quad f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Show that $f(x)$ is differentiable at $x = 0$ and $f'(0) = 0$.