Maxima and Minima

1. Determine two positive numbers whose sum is 15 and the sum of whose squares is minimum. [**Ans. :** 15/2, 15/2]

2. Divide 64 into two parts such that the sum of the cubes of two parts is minimum. [**Ans. :** 32, 32]

3. Divide 15 into two parts such that the square of one multiplied with the cube of the other is minimum. [**Ans. :** 6, 9]

4. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.

5. Show that among all positive numbers x and y with $x^2 + y^2 = r^2$, the sum x + y is largest when x = y = $r/\sqrt{2}$. $\left[\textbf{Ans.}: height = \frac{20}{\pi+4}m,\ width = \frac{10}{\pi+4}m\right]$

6. The space *s* described in time *t* by a particle moving in a straight line is given by $S = t^5 - 40t^3 + 30t^2 + 80t - 250$. Find the minimum value of acceleration. [**Ans. :** a = –260 at t = 2]

7. A particle is moving in a straight line such that its distance *s* at any time *t* is given by $S = \frac{t^4}{4} - 2t^3 + 4t^2 - 7$. Find when its velocity is maximum and acceleration minimum.

   [**Ans. :** Velocity is maximum at $t = 2 - \frac{2}{\sqrt{3}}$, acceleration is minimum at t = 2]

8. Prove that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

9. If $f(x) = x^3 + ax^2 + bx + c$ has a maximum at x = – 1 and minimum at x = 3. Determine a, b and c. [**Ans. :** a = –3, b = –9, c ∈ R]

10. A closed cylinder has volume 2156 cm^3. What will be the radius of its base so that its total surface area is minimum. [**Ans. :** 7 cm]

11. Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm long. $\left[\textbf{Ans.}: \frac{25}{4}cm^2\right]$