IB Elite Tutor

Q.1. a single dice rolled 36 times probability is given by

Score: 1,2,3,4,5,6

Frequency: 3,5,4,6,10,8

Mean = (3x1+5x2+4x3+6x4+10x5+8x6)/(3+5+4+6+10+8)

=4.08

Standard deviation=

$$\sqrt{(3.(1-4.08)^2+5.(2-4.08)^2+4(3-4.08)^2+6(4-4.08)^2+10(5-4.08)^2+8(6-4.08)^2}$$

=1.60

(ii) median = (n+1)th/2 term=7/2

3.5th term

Median = 5.5

(iii) interquartile range = upper quartile – lower quartile

$$Q_1 = l + \left(\frac{\frac{n}{4} - pcf}{f}\right) i \ , \ Q_3 = l + \left(\frac{\frac{3n}{4} - pcf}{f}\right) i$$

$Q_1 = 3.2,$ $Q_{3=5.9}$ Interquartile range = Q3-Q1=2.7

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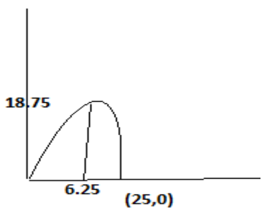
Q.2 $v(t) = 15\sqrt{t} - 3t$



Which is quadratic in nature.

$$\frac{dv}{dt} = 15\left(\frac{1}{2}\right)\frac{1}{\sqrt{t}} - 3 = 0$$

$$=> \sqrt{t} = \frac{5}{2} => t = 6.25$$

Also for v=0 i.e. $15\sqrt{t} - 3t = 0 => 9t^2 = 225t => t = 0 \; or \; t = \frac{225}{9} = 25$

B(i) $d(t) = \int_{t=a}^{t=b} \left(15\sqrt{t} - 3t\right)dt$

B(ii) d(9)= $\int_{t=0}^{t=9}\left(15\sqrt{t} - 3t\right)dt => \left[\frac{2}{3}15t^{\frac{3}{2}} - \frac{3}{2}t^2\right] \; with \; limit \; 0 \; to \; 9$

$$=> 148.5 \; m$$

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Q.3. as nth term of A.P series is $a_n = a + (n-1)d$

(i) $u_n = 5 + 2n$

At n=1 , $u_1 = 5 + 2.1 => u_1 = 7$

At n=2, $u_2 = 5 + 2.2 => u_2 = 9$

At n=3,$u_3 = 5 + 2.3 => u_1 = 11$

Therefore series are 7,9,11.......

Such that d=2

(ii) given nth term =115,

As $u_n = 5 + 2n$

=> $115 = 5 + 2n => n = \frac{115-5}{2} => n = 55$

(iii) sum of the series $S_n = \frac{n}{2}$(2a+(n-1)d)

As given no. Of terms are 55 such that

$S_{55} = \frac{55}{2}$(2a+(55-1)d)

$S_{55} = \frac{55}{2}$(2(7)+(55-1)2)=> $S_{55} = 3355$

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Q.4 given $v = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$

$$|v| = \left|\sqrt{2^2 + (-3)^2 + 6^2}\right|$$

$= \left|\sqrt{4 + 9 + 36}\right| => 7$

$$\frac{\vec{v}}{|v|} = \begin{pmatrix} 2/7 \\ -3/7 \\ 6/7 \end{pmatrix}$$

Similarly $|w| = \left|\sqrt{k^2 + 4 + 16}\right| => \left|\sqrt{k^2 + 20}\right|$

$$\frac{\vec{w}}{|w|} = \begin{pmatrix} k/\sqrt{k^2+20} \\ -2/\sqrt{k^2+20} \\ 4/\sqrt{k^2+20} \end{pmatrix}$$

Then $\cos\theta = \frac{\vec{v}}{|v|}.\frac{\vec{w}}{|w|}$

$$=> \frac{2k + 6 + 24}{7\sqrt{k^2 + 20}} => \cos\frac{\pi}{3}$$

$$\frac{2k + 30}{7\sqrt{k^2 + 20}} = \frac{\sqrt{3}}{2}$$

$$=> \frac{(2k + 30)^2}{49(k^2 + 20)} = \frac{3}{4}$$

$$=> k = 18.8$$

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Q.5 given $f(x) = px^3$+q$x^2 + rx$

$$f(-2) = -8 = -8p + 4q - 2r \dots\dots\dots\dots\dots\dots\dots (1)$$

$$f(1) = -2 = p + q + r \dots\dots\dots\dots\dots\dots\dots..(2)$$

$$f(2) = 0 = 8p + 4q + 2r \dots\dots\dots\dots\dots\dots..(3)$$

Then we have these solve by add equation 1 and 3 we get $8q = -8 \;=> q = -1$

Solve from equation 2 we get $p - 1 + r = -2 => p + r = -1$

Than we have $8p - 4 = 2r = 0 => 8p = 2r = 4 => 4p + r = 2$

Now solve $p + r = -1 \; and \; 4p + r = 2$

We get $p = 1, r = -2$

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Q6. Area of triangle $A = ab\sin\frac{\theta}{2}$

Given that area=20, side AC= 5cm, side BC=13.6

Let side AC=a, side BC=b

Such that $\theta = \sin^{-1}\frac{2A}{ab}$

$$=> \ \theta = \sin^{-1}\frac{2(20)}{(5)(13.6)}$$

$$\theta = \sin^{-1}(0.58) => there\ are\ two\ soutions\ \theta = 36.03(in\ degree\ )\ or\ \theta = 143.9$$

Therefore $A\grave{C}B = 144^o$

(ii) Area of triangle by herons formula

$$A = \sqrt{S(S-a)(S-b)(S-c)}$$

Can be write as $C = \sqrt{a^2 + b^2 + 2\sqrt{(ab)^2 - 4A^2}}$

$$C = \sqrt{5^2 + (13.6)^2 + 2\sqrt{(5(13.6))^2 - 4(20)^2}}$$

$$C = \sqrt{5^2 + (13.6)^2 + 2\sqrt{4624 - 1600}}$$

$$C = \sqrt{25 + 184.9 + 109.9}$$

$$C = 17.88$$

Thus side AC=17.8

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Q.7 given $f'(x) = -24x^3 + 9x^2 + 3x + 1$

(a) point of inflexion of f(x) are maxima or minima points of F'(x) so we will find maxima or minima of F'(x)

Then f''(x)= $= -72x^2 + 18x + 3 = 0$

$$=> x = \frac{-18 \pm \sqrt{18^2 + 4(72)(3)}}{2(-72)} => -0.115\ and\ 0.365$$

$$f'''(x) = -144x = 18$$

$$f'''(-0.115) = 34.56 > 0\ ie. -0.115\ is\ minnma\ of\ f'(x)$$

$$f'''(0.365) = -34.56 < 0\ ie. 0.365\ is\ maxima\ of\ f'(x)$$

Then -0.115 and 0.365 are point of inflexion of f(x)

(b) $g(x) = F''(x) = -72x^2 + 18x + 3$

Which is quadratic

Then $f'''(x) = -144x + 18 \; which \; is \; linear$

Now max. Or min of f'''(x) would be point of inflexion of f''(x) . since f'''(x) has no max.or minima

F'''(x) has not point of inflexion.

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Q.8(a) $f(x) = x\log(4 - x^2)$

Then $x\log(4 - x^2) = 0$

When x not equal to 0 => $\log(4 - x^2) = 0 => 4 - x^2 = 0 => x = \pm\sqrt{3}$

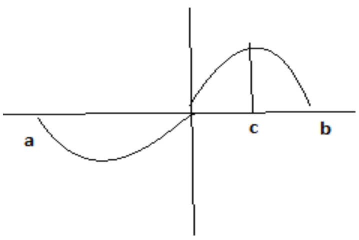
(b) $f'(x) = \log(4 - x^2) + x\left(\frac{1}{4-x^2}\right)(-2x)$

=> $\log(4 - x^2) + x\left(\frac{1}{4-x^2}\right)(-2x) = 0$

=> $\log(4 - x^2) = x\left(\frac{1}{4-x^2}\right)(2x)$

$=> x = 1.15$

(d)



Area A=$\left|\int_a^0 x\log(4 - x^2)\,dx\right| + \left|\int_0^c x\log(4 - x^2)\,dx\right|$

Now $\int_a^0 x\log(4 - x^2)\,dx$ we put 4-$x^2 = 0 => -2xdx = dt$

Then $\int(-1/2)\; logt\; dt = -1/2\int 1.logt\; dt => -1/2\left[logt.t - \int\frac{1}{t}.tdt\right]$

=$\frac{-1}{2}t(logt - 1)$

Using integration putting limits $\int_a^0 x\log(4-x^2)\,dx = \frac{-1}{2}(4-x^2)(\log(4-x^2)-1) atlimits\ x = -\sqrt{3}\ to\ 0$

And similarly $\int_0^c x\log(4-x^2)\,dx = \frac{-1}{2}(4-x^2)(\log(4-x^2)-1) atlimits\ x = 0\ to\ 1.15$

Adding above =2.07

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Q.9 In this question belongs to binomial distribution

(i) $p = \frac{1}{5}\ \ (winning), q = \frac{4}{5}\ \ (losing)$

$$E(x) = np = \frac{5.1}{5} = 1$$

(ii) to pass the exam can solve 3 or 4or5 question correctly i.e.

$$5_{C_3 p^3 q^2 + 5_{C_4} p^4}\ q + 5_{C_5} p^5$$

$$=> 10\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 + 5\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^1 + \left(\frac{1}{5}\right)^5$$

$$=> \frac{181}{5^5} => 0.057$$

B(i) since $\sum p_i = 1$

$$=> \ 0.67 + 0.05 + a + 2b + a - b + 2a + b + 0.04 = 1$$

$$=> 4a + 2b = 0.24$$

B(ii) $E(x) = \sum X_i\, p_i$

$$=> 0(0.67) + 1(0.05) + 2(a + 2b) + 3(a - b) + 4(2a + b) + 5(0.04) = 1$$

$$=> 0.05 + 2a + 4b + 3a - 3b + 8a + 4 + 0.2 = 1$$

$$=> 13a + 5b = 0.75$$

Solve $4a + 2b = 0.24$ and $13a + 5b = 0.75$

We get $a = 0.05, b = 0.02$

(c) Probability of bill passing exam is

$$P(X = 3) + P(X = 4) + P(X = 5)$$

$$=> (a - b) + (2a + b) + (0.04)$$

$$=> (0.05 - 0.02) + 0.10 + 0.02 + 0.04$$

$$=> 0.19(> 0.0579\ of\ marks)$$

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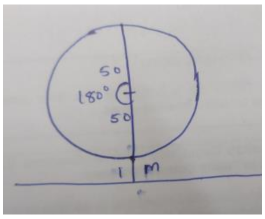
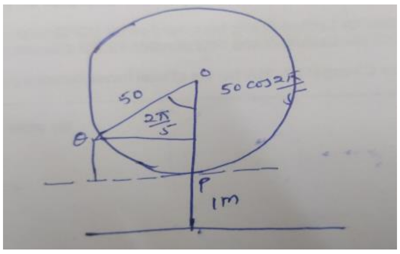
Q10. (a) 30 minutes the wheel goes 360 (=2π)

$$=> 15\ minutes\ the\ wheel\ goes\ 180$$

So height 50+50+1=101 

(b) in 6 minutes the wheel rotates by $\frac{2\pi}{30}\ 6\ radians$

=$\frac{2\pi}{5}\ radians$

=50(0.309)=15.45 

Than 50-15.45=34.54

Such that answer is 34.54+1=35.54

(c) $h(t) = 50\sin\big(b(t-c)\big) + 51$

Given when t=0, h(0)=1, when t=15, h(15)=101

Since two unknowns we require two conditions

$$h(0) = 1 = 50\sin(-bc) + 51 => bc = \frac{\pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$h(15) = 101 = 50\sin\big(b(15-c)\big) + 51 => 15b - bc = \frac{\pi}{2} \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

solve (1) and (2) we get $b = \frac{\pi}{15}, c = \frac{15}{2}$

(d) the models say $h(t) = 50\sin\left(\frac{\pi}{15}\left(t-\frac{15}{2}\right)\right) + 51$

When h(t)=96 we have 96=$50\sin\left(\frac{\pi}{15}\left(t-\frac{15}{2}\right)\right) + 51$

$$=> \frac{45}{50} = \sin\left(\frac{\pi}{15}\left(t-\frac{15}{2}\right)\right)$$

$$=> \sin^{-1}(0.9)\left(\frac{15}{\pi}\right)+\frac{15}{2}=t$$

$$=> t = 12.84m$$

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