

Limits

Find the value of following:

1. $\lim_{x \rightarrow 1} \frac{\sqrt{1 - \cos 2(x-1)}}{x-1}$.

[Ans. : does not exist because left hand limit is not equal to right hand limit.]

2. $\lim_{x \rightarrow \pi/2} \tan^2 x \left(\sqrt{2 \sin^2 x + 3 \sin x + 4} - \sqrt{\sin^2 x + 6 \sin x + 2} \right)$.

[Ans.: $\frac{1}{12}$]

3. $\lim_{x \rightarrow 0} \frac{1 - \cos(1 - \cos x)}{x^4}$.

[Ans.: $\frac{1}{8}$]

4. $\lim_{x \rightarrow \infty} \left(\frac{3x-4}{3x+2} \right)^{\left(\frac{x+1}{3} \right)}$.

[Ans.: $e^{-2/3}$]

5. $\lim_{x \rightarrow 0} \left(\frac{1+5x^2}{1+3x^2} \right)^{1/x^3}$.

[Ans.: e^2]

6. $\lim_{h \rightarrow 0} \frac{\ln(1+2h) - 2\ln(1+h)}{h^2}$.

[Ans. : - 1]

7. $\lim_{x \rightarrow 1} (\log_5 5x)^{\log_x 5}$.

[Ans. : e]

8. $\lim_{x \rightarrow 1} (\log_2 2x)^{\log_x 5}$.

[Ans.: $e^{\log_2 5}$]

9. $\lim_{x \rightarrow \infty} \sqrt{a^2 x^2 + ax + 1} - \sqrt{a^2 x^2 + 1}$.

[Ans.: $\frac{1}{2}$]

10. $\lim_{x \rightarrow 0} \frac{x(5^x - 1)}{1 - \cos x}$.

[Ans. : 2 log 5]

11. If $f(x) = \left(\frac{x^2 + 5x + 3}{x^2 + x + 2} \right)^x$ then $\lim_{x \rightarrow \infty} f(x)$. [Ans. : e^4]

12. $\lim_{x \rightarrow 0} \left(\frac{\sin x - x + \frac{x^3}{6}}{x^5} \right)$. [Ans. : $\frac{1}{120}$]

13. $\lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \log(1-x)}{x^3}$. [Ans. : $-\frac{1}{2}$]

14. $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x} \right)^x$.

[Ans. : e^2]

15. $\lim_{x \rightarrow 0} \frac{x \cos x - \log(1+x)}{x^2}$. [Ans. : $\frac{1}{2}$]

16. $\lim_{x \rightarrow \infty} \left(\frac{x+2}{x+1} \right)^{x+3}$. [Ans. : e]

17. $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$. [Ans. : -1]

18. $\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4}$. [Ans. : e^5]

19. $\lim_{x \rightarrow 1} \frac{1 + \log x - x}{1 - 2x + x^2}$. [Ans. : none of these]

20. Constants a and b so that $\lim_{x \rightarrow \infty} \left(\frac{x^2 + 1}{x + 1} - ax - b \right) = 0$. [Ans. : $a = 1, b = -1$]

21. $\lim_{x \rightarrow -1} \frac{\sqrt{\pi} - \sqrt{\cos^{-1} x}}{\sqrt{x+1}}$. [Ans. : $\frac{1}{\sqrt{2\pi}}$]