

Maxima and Minima

1. Determine two positive numbers whose sum is 15 and the sum of whose squares is minimum. [Ans. : 15/2, 15/2]
2. Divide 64 into two parts such that the sum of the cubes of two parts is minimum. [Ans. : 32, 32]
3. Divide 15 into two parts such that the square of one multiplied with the cube of the other is minimum. [Ans. : 6, 9]
4. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$.
5. Show that among all positive numbers x and y with $x^2 + y^2 = r^2$, the sum x + y is largest when $x = y = r/\sqrt{2}$. [Ans.: height = $\frac{20}{\pi+4}m$, width = $\frac{10}{\pi+4}m$]
6. The space s described in time t by a particle moving in a straight line is given by $S = t^5 - 40t^3 + 30t^2 + 80t - 250$. Find the minimum value of acceleration. [Ans. : a = -260 at t = 2]
7. A particle is moving in a straight line such that its distance s at any time t is given by $S = \frac{t^4}{4} - 2t^3 + 4t^2 - 7$. Find when its velocity is maximum and acceleration minimum. [Ans. : Velocity is maximum at $t = 2 - \frac{2}{\sqrt{3}}$, acceleration is minimum at t = 2]
8. Prove that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.
9. If $f(x) = x^3 + ax^2 + bx + c$ has a maximum at $x = -1$ and minimum at $x = 3$. Determine a, b and c. [Ans. : a = -3, b = -9, c ∈ R]
10. A closed cylinder has volume 2156 cm³. What will be the radius of its base so that its total surface area is minimum. [Ans. : 7 cm]
11. Find the largest possible area of a right angled triangle whose hypotenuse is 5 cm long. [Ans.: $\frac{25}{4}cm^2$]