IB Elite Tutor

Solutions question paper -1

Q.1. Three term are of a geometric series are in G.P. then say G.P. is a, ar,ar^2.....

Than general term $u_n = ar^{n-1}$

As a =32 and common ratio is $r = \frac{ar}{a} = \frac{16}{32} = \frac{1}{2}$

Such that $u_6 = 32(1/2)^{6-1}$

$$u_6 = 1$$

Now $S_n = a + ar + ar^2 \ldots \ldots\ldots$, when $|r| < 1$ is given as

$$S_n = \frac{a}{1-r} = 64$$

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Q.2 let $g(x) = 2xsinx$

$$\frac{d(cf(x))}{dx} = c\frac{df(x)}{dx}$$

And $\frac{df(x)h(x)}{dx} = \frac{df(x)}{dx}h(x) + f(x)\frac{dh(x)}{dx}$

$$\frac{d2xsin(x)}{dx} = \frac{d(2x)}{dx}sin(x) + (2x)\frac{dsin(x)}{dx}$$

$$= 2sinx + 2xcosx$$

At $x = \pi$, $\frac{d2xsin(x)}{dx} = 2sin\pi + 2\pi cos\pi = -2\pi$

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Q3. $S = r\theta$ (θ $in\ radian$)

(a) $S = 10.\frac{\pi}{3}$

$$area\ of\ sector = \pi r^2\frac{\theta}{2\pi}$$

(b) area of shaded portion is $\pi r^2\frac{\theta}{2\pi} = \pi(100)\frac{\frac{\pi}{3}}{2\pi} = \frac{\pi^2}{6\pi}(100 - 64) = 6\pi$

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Q4 (a) P(A)+P(A')=1 according to the rule or probability

Given that P(A) =1/5

Such that P(A')=1-1/5=4/

(b) P(B)=P(A$\cap B) + P(A' \cap B)$

$=P\left(\frac{B}{A}\right)P(A) + P\left(\frac{B}{A'}\right)P(A') = \frac{1}{4}\frac{1}{5} + \frac{3}{8}\frac{4}{5} = \frac{14}{40} = \frac{7}{20}$

(C)By using baye's theorem

$$P\left(\frac{A'}{B}\right) = \frac{P\left(\frac{B}{A'}\right)P(A')}{P(B)} = \frac{\frac{3}{8}\cdot\frac{4}{5}}{\frac{7}{20}} = \frac{6}{7}$$

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Q.5 using property $cos2\theta = 1 - 2\,sin^2\theta$

$$4 - (1 - 2sin^2\theta) + 5sin\theta = 4 - 1 + 2sin^2\theta + 5sin\theta$$

$=2sin^2\theta + 5sin\theta + 3$

Now $2sin^2\theta + 5sin\theta + 3 = 0$

Let sin$\theta = x$

Such that $2x^2$+5x+3=0

Roots are $x = \frac{-5\pm\sqrt{25-4.2.3}}{4} = \frac{-5\pm1}{4} = \frac{-3}{2}, -1$

So when $x = \frac{-3}{2}, than\ sin\theta = -\frac{3}{2}$

Since $|sin\theta| \leq 1, such\ that\ sin\theta = -\frac{3}{2} is\ not\ possible.$

When x=-1, $sin\theta = -1, 0 \leq \theta \leq 2\pi$

Hence $\theta = \frac{3\pi}{2}$

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Q.6 given that $\frac{df(x)}{dx}$ $=sin\ sin\ (2x-3)$

Using separation of variables

$$\int d(f(x) = \int sin\, sin\, (2x-3)\, dx$$

$$f(x) = -\,cos\, cos\, (2x-3)/2\ + c$$

Given x=3/2 and f(3/2)=4

$$4 = \frac{-\,cos\, cos\, \left(\frac{2.3}{2} - 3\right)}{2} + c$$

$$4 = -\frac{cos\, cos\, (0)}{2} + c$$

$$c = 4 + \frac{1}{2}$$

$$c = \frac{9}{2}$$

Then $f(x) = \frac{-cos(2x-3)}{2} + \frac{9}{2}$

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Q7. $A = [9e^{x}\ e^{x}\ e^{x}\ e^{3x}\ ]$

DetA=$|9e^{x}\ e^{x}\ e^{x}\ e^{3x}\ |$=$|9e^{x}\ e^{x}\ e^{x}\ e^{x}e^{2x}\ |$

Using property of determinant

$$|9e^{x}\ e^{x}\ e^{x}\ e^{x}e^{2x}\ | = e^{x}e^{x}|9\ 1\ 1\ e^{2x}\ | = e^{2x}[9e^{2x} - 1]$$

Now A^{-1} is not possible is A is singular

i.e$|A| = 0 => e^{2x}[9e^{2x} - 1] = 0$

this means $e^{2x} = 0\ \ and\ \ 9e^{2x} - 1 = 0$

when $e^{2x} = 0 => 2xloge = log0 => x \to -\infty$

when $9e^{2x} - 1 = 0 => 9e^{2x} = 1 =>> e^{2x} = \frac{1}{9}$

$$=> 2xloge = log\, log\, \left(\frac{1}{9}\right) => 2x = log1 - log9 => x = -\frac{1}{2}log9$$

$$=> x = -\frac{1}{2}log3^{2}$$

$$=> x = (-1)log3$$

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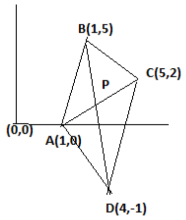
Q.8 from the figure



A(i)

$$\overrightarrow{OA} + \overrightarrow{AC} = \overrightarrow{OC}$$

$$(1\ 0\ ) + \overrightarrow{AC} = (5\ 2\ )$$

$==\overrightarrow{AC} = (5\ 2\ ) - (1\ 0\ ) => (4\ 2\ )$

Similarly $\overrightarrow{OB} + \overrightarrow{BD} = \overrightarrow{OD}$

$$(1\ 5\ ) + \overrightarrow{BD} = (4\ \ -1\ )$$

$$\overrightarrow{BD} = (4\ \ -1\ ) - (1\ 5\ ) => (3\ \ -6\ )$$

A(iii) $\overrightarrow{AC}.\overrightarrow{BD} = (4\ 2\ ).(3\ \ -6\ ) => 12 + (-12) => 0$

B(i) line $\overrightarrow{AC}$ is given as $r = (1\ 0\ ) + s(4\ 2\ )$

Were $(1\ 0\ )\ is\ \overrightarrow{OA}\ \ and\ (4\ 2\ )\ is \overrightarrow{AC}\ ; s\ is\ parameter$

B(ii) similarly $\overrightarrow{BD} = (1\ 5\ ) + t(3\ \ -6\ ), where\ t\ is\ parameter$

(c)

Lines AC and BD interest at point P

$(1\ 0\ ) + s(4\ 2\ ) = (1\ 5\ ) + t(3\ \ -6\ )$ at point of intersection co-ordinates are same

$=>> (4s\ 2s\ ) - (3t\ \ -6t\ ) = (0\ 5\ ) => 4s - 3t = 0, 2s + 6t = 5 => t = \frac{2}{3}, s = \frac{1}{2}$

When s=1/2 the point must be P

$$r = (1\ 0\ ) + s(4\ 2\ )$$

$$i.e\ (x\ y\ ) = (1\ 0\ ) + s(4\ 2\ ) => (3\ 1\ )$$

Given P(3,k) then k=1

(d) now $\overrightarrow{PD \amalg \quad AC}$

Area of triangle ACD=1/2.$|PD||AC|$

$$|PD| = \left|\sqrt{(3-4)^2 + (1-(-1))^2}\right| => \sqrt{5}$$

$$|AC| = \left|\sqrt{(1-5)^2 + (0-2)^2}\right| => \sqrt{20}$$

Area =1/2. $\sqrt{5}$. $\sqrt{20}$=5

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Q.9 f(x)=$x^2 + 4, g(x) = x - 1$

(a) (fog)(x)=f(g(x))=f(x-1)=$(x-1)^2 + 4 = x^2 - 2x + 5$

Also fog(x) =$(x-1)^2 + 4$ i.e vertex is (1,4)

Vector $3 \ \ -1$ translate the graph i.e $x = X - 3, y = Y - (-1)$

Then $(x \ y) = (X \ Y) + (-3 \ 1)$

$$=> (1 \ 4) = (X \ Y) + (-3 \ 1) => (X \ Y) = (4 \ 3)$$

(b) then co=ordinates of vertex of h are (4,3)

Then h(x)=$(x-4)^2 + 3$

$$=> h(x) = x^2 - 8x + 19$$

(c) line y=2x-6 is tangent tograph h. Slope of line is 2

Gradient of h(x) is $h'(x) = 2x - 8$

When is gradient = slope

i,e, 2x-8=2

=> $x = 5$ **(co-ordinate of P)**

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Q.1O $f(x) = x^3, f'(x) = 3x^2$

Now PQ is a line passing through (a,f(a))and (2/3,0)

i.e. (a, a^3) and (2/3,0)

then line has slope which is gradient

(i)$\frac{a^3-0}{a-2/3} = \frac{a^3}{a-2/3}$

(ii) $f'(x) = 3x^2$=> $f'(x) = 3a^2$

Which is gradient of tangent at point (a,f(a)) is same as line PQ

So $3a^2 = \frac{a^3}{a-2/3}$

$$=> 3a^2\left(a - \frac{2}{3}\right) = a^3$$

$$=> 3a - 2 = a => 2a = 2 => a = 1$$

As per the diagram Area A of region T is given as

$$A = \int_{x=-2}^{x=k} \{(f(x) - (3x - 2)\}dx$$

$A = \int_{x=-2}^{x=k} \{x^3 - (3x - 2)\}dx$

$$A = \int_{x=-2}^{x=k} \{x^3 - 3x + 2)\}dx$$

$A = \frac{x^4}{4} - 3\frac{x^2}{2} + 2]$ at limit x=-2 to k

Then given that

$$\left\{\frac{k^4}{4} - 3\frac{k^2}{2} + 2k\right\} - \left\{\frac{(-2)^4}{4} - 3\frac{(-2)^2}{2} + 2(-2)\right\} = 2k + 4$$

$$\left\{\frac{k^4}{4} - 3\frac{k^2}{2} + 2k + 6\right\} = 2k + 4$$

$$=> \frac{k^4}{4} - 3\frac{k^2}{2} + 2 = 0$$

$$=> k^4 - 6k^2 + 8 = 0$$

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